A Class of Blind Deconvolution and Equalization Algorithms for Nonminimum Phase Multi-Input Multi-Output Systems

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Abstract

A unified class of inverse filter criteria using two cumulants, which includes Wiggins' criterion, Donoho's criteria and Tugnait's criteria as special cases, has been proposed by Chi and Wu for blind deconvolution and equalization of real single-input single-output (SISO) linear time-invariant (LTI) systems. In this paper, we extend this class of (single channel) inverse filter criteria to a family of multistage and a family of single stage criteria for deconvolution and equalization of real (or complex) multi-input multi-output (MIMO) LTI systems with only non-Gaussian measurements. It can be shown that the two families of inverse filter criteria lead to perfect equalization for MIMO systems under some conditions. Some simulation results for the optimum inverse filter using gradient type iterative optimization algorithms were provided to support the proposed criteria. Finally, we draw some conclusions.

1. Introduction

Multichannel blind deconvolution and equalization is a problem to estimate a desired signal $\mathbf{u}(n) = [u_1(n), \ldots, u_p(n)]^{\mathrm{T}}$ with only a set of measurements $\mathbf{x}(n) = [x_1(n), \ldots, x_q(n)]^{\mathrm{T}}$ given by the following convolutional model:

$$\mathbf{x}(n) = \mathbf{H}(n) * \mathbf{u}(n) = \sum_{k=-\infty}^{\infty} \mathbf{H}(k)\mathbf{u}(n-k) \quad (1)$$

where $\mathbf{H}(n)$ is the $q \times p$ impulse response matrix sequence of a p-input q-output linear time-invariant (LTI) system. The problem has recently drawn extensive attention in wireless communications, such as mobile communications with asynchronous direct-sequence code-division multiple access (DS-CDMA), data communications over dually polarized multipath channel, and array signal processing for base-station with spatial division multiple access (SDMA).

Higher-order statistics (HOS) and inverse filters have been used for multichannel blind deconvolution and equalization [1-5]. Inouye and Sato [2] extended Shalvi and Weinstein's (single channel) inverse filter criterion [6] to both multistage (MS) and single stage (SS) multichannel inverse filter criteria. Tugnait [3] also extended the single channel inverse filter criteria reported in [7] to MS multichannel inverse filter criteria. These criteria use only second- and third- or fourth-order cumulants of signals. On the other hand, Chi and Wu [8] proposed a class of single channel inverse filter criteria $J_{r,m}$ using an rth-order (even) and an mth-order ($> r$) cumulants of real data. This class includes Wiggins' criterion (associated with $J_{2,4}$) [9], Donoho's criteria (associated with $J_{2,m}$) [10], and Tugnait's (single channel) criteria $J_{2,3}$, $J_{2,4}$ and $J_{4,6}$ [7] as special cases. Note that Shalvi and Weinstein's criteria [6] are actually the same as Donoho's criteria for real data. In this paper, we extend this class of inverse filter criteria $J_{r,m}$ to a family of MS and a family of SS inverse filter criteria for real (or complex) multi-input multi-output (MIMO) LTI systems.

2. Model assumptions and problem formulation

Assume that $\mathbf{x}(n)$, $n = 0, \ldots, N-1$ are a given set of real (or complex) non-Gaussian measurements generated from (1) with the following assumptions:

(A1) The components $u_i(n)$, $i = 1, \ldots, p$, of the input vector $\mathbf{u}(n)$ are real (or complex), zero-mean,

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i.i.d. non-Gaussian, and all of them are statistically independent of each other.

(A2) The unknown p-input q-output LTI system

$$\mathcal{H}(z) = \sum_{n=-\infty}^{\infty} \mathbf{H}(n) z^{-n} \qquad (2)$$

is real (or complex) stable with possibly nonminimum phase.

(A3) $q \geq p$.

(A4) The MIMO LTI system $\mathcal{H}(z)$ is of full rank on the unit circle, i.e., $rank\{\mathcal{H}(z)\} = p$ for $|z| = 1$.

As mentioned in [3], the assumptions (A3) and (A4) are a set of sufficient conditions for the existence of the stable inverse filter of $\mathcal{H}(z)$.

Assume that $\mathcal{V}(z)$ is a stable $p \times q$ MIMO LTI system and $\mathbf{V}(n)$ is the impulse response matrix sequence of $\mathcal{V}(z)$. Let

$$\begin{aligned} \mathbf{e}(n) &= [e_1(n), \ldots, e_p(n)]^{\mathrm{T}} \\ &= \mathbf{V}(n) * \mathbf{x}(n) = \mathbf{G}(n) * \mathbf{u}(n) \end{aligned} \qquad (3)$$

where

$$\mathbf{G}(n) = \mathbf{V}(n) * \mathbf{H}(n) \quad \text{(see (1))} \qquad (4)$$

is the $p \times p$ impulse response matrix sequence of the combined overall system, denoted $\mathcal{G}(z)$.

With $\mathcal{V}(z)$ as an estimate for $\mathcal{H}(z)$'s inverse system, the goal of multichannel deconvolution and equalization is to find an optimum $\mathcal{V}(z)$ such that

$$\mathcal{G}(z) = \mathcal{V}(z) \cdot \mathcal{H}(z) = \mathbf{P} \cdot \mathcal{D}(z) \qquad (5)$$

(perfect equalization) where $\mathbf{P}$ is a (nonsingular) permutation matrix and $\mathcal{D}(z)$ is a $p \times p$ diagonal matrix given by

$$\mathcal{D}(z) = \mathrm{diag}(\alpha_1 z^{-\tau_1}, \cdots, \alpha_p z^{-\tau_p}) \qquad (6)$$

in which α_i, $i = 1, \ldots, p$ are unknown real (or complex) scale factors and τ_i, $i = 1, \ldots, p$ are unknown time delays. Consequently, it can be easily observed from (3) and (5) that the optimum equalized signal

$$\mathbf{e}(n) = [\alpha_{i_1} u_{i_1}(n - \tau_{i_1}), \ldots, \alpha_{i_p} u_{i_p}(n - \tau_{i_p})]^{\mathrm{T}} \qquad (7)$$

where $\{i_1, \ldots, i_p\}$ is a permuted sequence of the sequence $\{1, \ldots, p\}$ associated with $\mathbf{P}$.

3. Inverse filter criteria for MIMO LTI systems

For ease of later use, let $h_{ij}(n)$, $v_{ij}(n)$ and $g_{ij}(n)$ denote the (i,j)th elements of the matrices $\mathbf{H}(n)$, $\mathbf{V}(n)$ and $\mathbf{G}(n)$, respectively. Moreover, let

$$C_{l_1,l_2}^{e_i} = \mathrm{CUM}(\underbrace{e_i(n), \ldots, e_i(n)}_{l_1 \text{ terms}}, \underbrace{e_i^*(n), \ldots, e_i^*(n)}_{l_2 \text{ terms}}) \qquad (8)$$

denote the $(l_1 + l_2)$th-order cumulant of the ith equalized signal ($i \in \{1, \ldots, p\}$)

$$e_i(n) = g_{i1}(n) * u_1(n) + \cdots + g_{ip}(n) * u_p(n) \qquad (9)$$

where the superscript '*' denotes complex conjugation.

The new multichannel inverse filter criteria to be presented below are based on the following theorem:

Theorem 1. Let the $(l_1 + l_2)$th-order cumulant of $u_i(n)$ be $\gamma_{l_1,l_2}^{u_i}$ where $i = 1, \ldots, p$. Assume that all the $(2s)$th-order cumulants of $u_i(n)$, $i = 1, \ldots, p$, have the same sign, i.e.,

$$\mathrm{sign}\{\gamma_{s,s}^{u_1}\} = \mathrm{sign}\{\gamma_{s,s}^{u_2}\} = \cdots = \mathrm{sign}\{\gamma_{s,s}^{u_p}\} \qquad (10)$$

Then under (A1) through (A4) and $l_1 + l_2 > 2s \geq 2$,

$$\widetilde{J}_i \triangleq \frac{|C_{l_1,l_2}^{e_i}|^{2s}}{|C_{s,s}^{e_i}|^{l_1+l_2}} \qquad (11)$$

$$\leq \kappa_{\max} \triangleq \max\{\kappa_j,\ j = 1, \ldots, p\} \qquad (12)$$

where

$$\kappa_j = \frac{|\gamma_{l_1,l_2}^{u_j}|^{2s}}{|\gamma_{s,s}^{u_j}|^{l_1+l_2}} \qquad (13)$$

The equality of (12) holds if and only if

$$g_{ij}(n) = \alpha_i \delta(n - \tau_i) \delta(j - j_0), \quad j_0 \in \{1, \ldots, p\} \qquad (14)$$

where α_i is an unknown real (or complex) scale factor, τ_i is an unknown time delay, and j_0 is the index associated with the maximum value of κ_j, $j = 1, \ldots, p$ (see (12)).

□

Note that the results presented in Theorem 1 for $(s, l_1, l_2) = (1, 2, 1)$ and $(1, 2, 2)$ have been proposed by Tugnait [3], while those for other choices of (s, l_1, l_2) such as $(1, 3, 1)$, $(1, 3, 2)$, ... are new. Next, based on Theorem 1, two families of multichannel inverse filter criteria, which follows, in part, the ideas proposed by Inouye and Sato [2], are presented for finding the optimum inverse filter $\mathbf{V}(n)$.

A. Family of MS Criteria

Let $\mathbf{v}_i(n)^{\mathrm{T}}$ denote the ith row vector of $\mathbf{V}(n)$. The family of MS criteria, which, at ith stage, try to find the optimum $\mathbf{v}_i(n)$ using $\widetilde{J}_i$ with some uncorrelatedness constraints on the obtained $(i-1)$ inverse filter output processes $e_1(n)$, ..., $e_{i-1}(n)$, is as follows:

Stage 1: Estimation of $\mathbf{v}_1(n)$.

Maximize

$$J_1^{(\mathrm{MS})} = \widetilde{J}_1 \quad (15)$$

where $\widetilde{J}_1$ is given by (11) and $l_1 + l_2 > 2s \geq 2$.

Stage i: Estimation of $\mathbf{v}_i(n)$ for $i \geq 2$.

Maximize

$$J_i^{(\mathrm{MS})} = \widetilde{J}_i - \lambda_i \sum_{k=1}^{i-1} \frac{\sum_{\tau \in Z} |C_{s,s}^{e_i e_k}(\tau)|^2}{|C_{s,s}^{e_i}||C_{s,s}^{e_k}|} \quad (16)$$

where $l_1 + l_2 > 2s \geq 2$, λ_i is a positive real constant, Z is the set of all integers, and

$$C_{s,s}^{e_i e_k}(\tau) = \mathrm{CUM}(\overbrace{e_i(n), \ldots, e_i(n)}^{s \text{ terms}}, \underbrace{e_k^*(n-\tau), \ldots, e_k^*(n-\tau)}_{s \text{ terms}}) \quad (17)$$

The deconvolution and equalization capabilities of the proposed MS criteria are supported by the following theorem:

Theorem 2. The MS criteria given by (15) and (16) lead to a solution for $\mathcal{G}(z)$ that satisfies (5) (perfect equalization), provided that λ_i given by (16) are chosen such that $\lambda_i \geq \kappa_{\max}$ (see (12)). □

B. Family of SS Criteria

Again, with some uncorrelatedness constraints on the inverse filter output processes, the family of SS criteria simultaneously estimates $\mathbf{v}_1(n)$, $\mathbf{v}_2(n)$, ..., $\mathbf{v}_p(n)$ by maximizing

$$J^{(\mathrm{SS})} = \sum_{i=1}^{p} \widetilde{J}_i - \lambda \sum_{i=2}^{p} \sum_{k=1}^{i-1} \frac{\sum_{\tau \in Z} |C_{s,s}^{e_i e_k}(\tau)|^2}{|C_{s,s}^{e_i}||C_{s,s}^{e_k}|} \quad (18)$$

where $l_1 + l_2 > 2s \geq 2$ and λ is a positive real constant, and meanwhile their deconvolution and equalization capabilities are supported by the following theorem.

Theorem 3. Under the assumption that all κ_i, $i = 1$, 2, , ..., p (see (13)), are the same, the SS criteria $J^{(\mathrm{SS})}$ lead to a solution for $\mathcal{G}(z)$ that satisfies (5) (perfect equalization). □

Two worthy remarks regarding the proposed MS and SS criteria are given as follows:

(R1) Theorems 2 and 3 are counterparts to the ones proposed by Inouye and Sato [2] for their MS and SS criteria, respectively, while the latter further requires a quite restrictive condition that $\gamma_{1,1}^{u_i} = 1$ for all i.

(R2) With $(s, l_1, l_2) = (1, 2, 1)$ or $(1, 2, 2)$, Tugnait's MS approach [3] obtains the equalized signal $e_i(n)$ as the estimate of an input signal $u_j(n)$ by unconstrained maximization of $\widetilde{J}_i$ (i.e., without any uncorrelatedness constraints on the inverse filter output processes) at the ith stage. However, this approach has to estimate the channel impulse responses $h_{lj}(n)$, $l = 1, \ldots, q$, for each stage i followed by removing the contribution of $u_j(n)$ from the measurements, leading to an MIMO system with q outputs and $(p - i)$ inputs for the next stage. On the other hand, besides many new choices for (s, l_1, l_2), the optimum inverse filter matrix $\mathbf{V}(n)$ is directly estimated by both of the proposed MS and SS criteria without need of estimation of the channel impulse responses for system dimension reduction.

4. Optimization algorithms for MS and SS criteria

To find the optimum inverse filter $\mathbf{V}(n)$ using the proposed MS and SS criteria given by (15), (16) and (18) with a given set of data, the cumulants used in these criteria can be simply replaced by the associated sample cumulants, and a causal FIR filter of order L can be used as an approximation to $\mathbf{V}(n)$. Since the objective function $J_i^{(\mathrm{MS})}$ given by (15) and (16) for stage i is a highly nonlinear function of

$$\mathbf{v}_i \triangleq [v_{i1}(0), \ldots, v_{i1}(L), \ldots, v_{iq}(0), \ldots, v_{iq}(L)]^{\mathrm{T}} \quad (19)$$

and the objective function $J^{(\mathrm{SS})}$ given by (18) is also a highly nonlinear function of

$$\mathbf{v} \triangleq [\mathbf{v}_1^{\mathrm{T}}, \mathbf{v}_2^{\mathrm{T}}, \ldots, \mathbf{v}_p^{\mathrm{T}}]^{\mathrm{T}} \quad (20)$$

We use gradient type iterative optimization algorithms to find the optimum $\mathbf{v}_i$. Specifically, at the kth iteration, $\mathbf{v}_i$ (associated with MS criteria) is updated by

$$\begin{cases} \mathbf{v}_i[k] = \mathbf{v}_i[k-1] + \rho \cdot \left. \dfrac{\partial J_i^{(\mathrm{MS})}}{\partial \mathbf{v}_i^*} \right|_{\mathbf{v}_i = \mathbf{v}_i[k-1]} \\ \mathbf{v}_i[k] = \mathbf{v}_i[k] / ||\mathbf{v}_i[k]|| \end{cases} \quad (21)$$

where ρ is a positive real constant. Note that the second line of (21) (i.e., normalization for $\mathbf{v}_i[k]$) is due to the fact that $J_i^{(\mathrm{MS})}$ is invariant to any scaled version of $\mathbf{v}_i$ (see (14)). For SS criteria, $\mathbf{v}$ given by (20) is also updated in a similar way except that $J_i^{(\mathrm{MS})}$ is replaced by $J^{(\mathrm{SS})}$ and normalization operation is performed for each $\mathbf{v}_i$ in $\mathbf{v}$ (see (20)).

5. Simulation results

In this section, let us present two simulation examples to demonstrate the proposed multichannel inverse filter criteria. In the two examples, synthetic noisy data $\mathbf{x}(n)$ for SNR = 20 dB were generated from (1), to which q uncorrelated white Gaussian noises were added.

Example 1. (Real signals)

Let us consider a real 2-input 2-output (*i.e.*, $p = q = 2$) MA(6) system $\mathcal{H}(z)$ (taken from [3]) given by

$$\begin{aligned} \mathcal{H}_{11}(z) &= 0.6455 - 0.3227z^{-1} + 0.6445z^{-2} \\ &\quad -0.3227z^{-3} \\ \mathcal{H}_{12}(z) &= 0.6140 + 0.3684z^{-1} \\ \mathcal{H}_{21}(z) &= 0.3873z^{-1} + 0.8391z^{-2} + 0.3227z^{-3} \\ \mathcal{H}_{22}(z) &= -0.2579z^{-1} - 0.6140z^{-2} + 0.8442z^{-3} \\ &\quad +0.4421z^{-4} + 0.2579z^{-6} \end{aligned}$$

The driving inputs $u_i(n)$, $i = 1, 2$, were assumed to be real, zero-mean, Exponentially i.i.d. with $\gamma_{1,1}^{u_i} = 1$, $\gamma_{2,1}^{u_i} = 2$, $\gamma_{2,2}^{u_i} = 6$, and $\gamma_{3,2}^{u_i} = 24$, $i = 1, 2$. The MS criteria for $(s, l_1, l_2) = (1, 2, 1)$, $(1, 2, 2)$ and $(1, 3, 2)$ were used to obtain the inverse filters $\mathbf{v}_i$, $i = 1, 2$, with filter order $L = 14$ and $\lambda_2 = \kappa_{\max}$ (i.e., $\lambda_2 = 4$, 36 and 576 for $(s, l_1, l_2) = (1, 2, 1)$, (1, 2, 2) and (1, 3, 2), respectively) as required by Theorem 2. Thirty independent runs for $N = 2048$ and 4096 were performed.

A performance index MISI defined as (taken from [1])

$$\mathrm{MISI} = \sum_{i=1}^{p} \frac{\sum_{j=1}^{p}[\sum_n |g_{ij}(n)|^2] - \max\{|g_{ij}(n)|^2, \forall j, n\}}{\max\{|g_{ij}(n)|^2, \forall j, n\}} + \sum_{j=1}^{p} \frac{\sum_{i=1}^{p}[\sum_n |g_{ij}(n)|^2] - \max\{|g_{ij}(n)|^2, \forall i, n\}}{\max\{|g_{ij}(n)|^2, \forall i, n\}} \quad (22)$$

was used as a measure of the multichannel intersymbol interference after equalization. Note that MISI = 0 when $\mathcal{G}(z)$ satisfies (5). Table 1 shows average values of MISI's, denoted $<$ MISI $>$, which were calculated with the obtained thirty independent estimates of $g_{ij}(n)$. One can see that all the MISI's after equalization are smaller than the MISI before equalization (the top row of Table 1) with the best MISI improvement (around 14 dB) for $(s, l_1, l_2) = (1, 2, 1)$.

Example 2. (Complex signals)

A real 2-input 2-output nonminimum-phase MA(2) system

$$\mathcal{H}(z) = \begin{bmatrix} 1 - 0.3z^{-1} + 0.8z^{-2} & -0.92z^{-1} \\ z^{-2} & 1 - 0.5z^{-1} + 0.2z^{-2} \end{bmatrix}$$

whose zeros are $0.5946 \pm j1.0738$ and $-0.1946 \pm j0.2614$, was used. The input $u_1(n)$ was assumed to be a 8-PSK signal of unity variance and the other input $u_2(n)$ was a 16-QAM signal of unity variance. The MS criteria for $(s, l_1, l_2) = (1, 2, 2)$ were used to obtain the inverse filters $\mathbf{v}_i$, $i = 1, 2$, with filter order $L = 20$ and $\lambda_2 = \kappa_{\max} = 1$ as required by Theorem 2. A single realization was performed for data length $N = 4096$.

The MISI before equalization is 7.8460 dB and the MISI after equalization is −13.6245 dB for this example, i.e., the use of the proposed MS criteria led to around 21 dB improvement in MISI. Moreover, Figures 1(a) and 1(b) show the unequalized signal constellations (i.e., eye patterns) associated with $x_1(n)$ and $x_2(n)$, respectively, for $n = 0 \sim 4095$. Figures 1(c) and 1(d) show the equalized signal constellations associated with $e_1(n)$ and $e_2(n)$, respectively, for $n = 0 \sim 4095$. One can see from these figures that the eye patterns after equalization are open to a sufficient degree.

6. Conclusions

Based on Theorem 1 and Inouye and Sato's ideas, we have extended Chi and Wu's single channel inverse filter criteria to a family of MS and a family of SS criteria for blind deconvolution and equalization of real (or complex) MIMO LTI systems. Furthermore, we proved (Theorems 2 and 3) that under some conditions, both of the proposed MS and SS criteria lead to perfect equalization, and their efficacy was justified through computer simulations.

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Table 1. Average values of MISI's over thirty independent runs for SNR = 20 dB.

Initial MISI = 9.8293 dB (before equalization)		
(s, l_1, l_2)	$<$ MISI $>$ (in dB)	
	$N = 2048$	$N = 4096$
(1,2,1)	−4.1371	−4.7832
(1,2,2)	0.4316	−2.4964
(1,3,2)	5.8262	2.3185

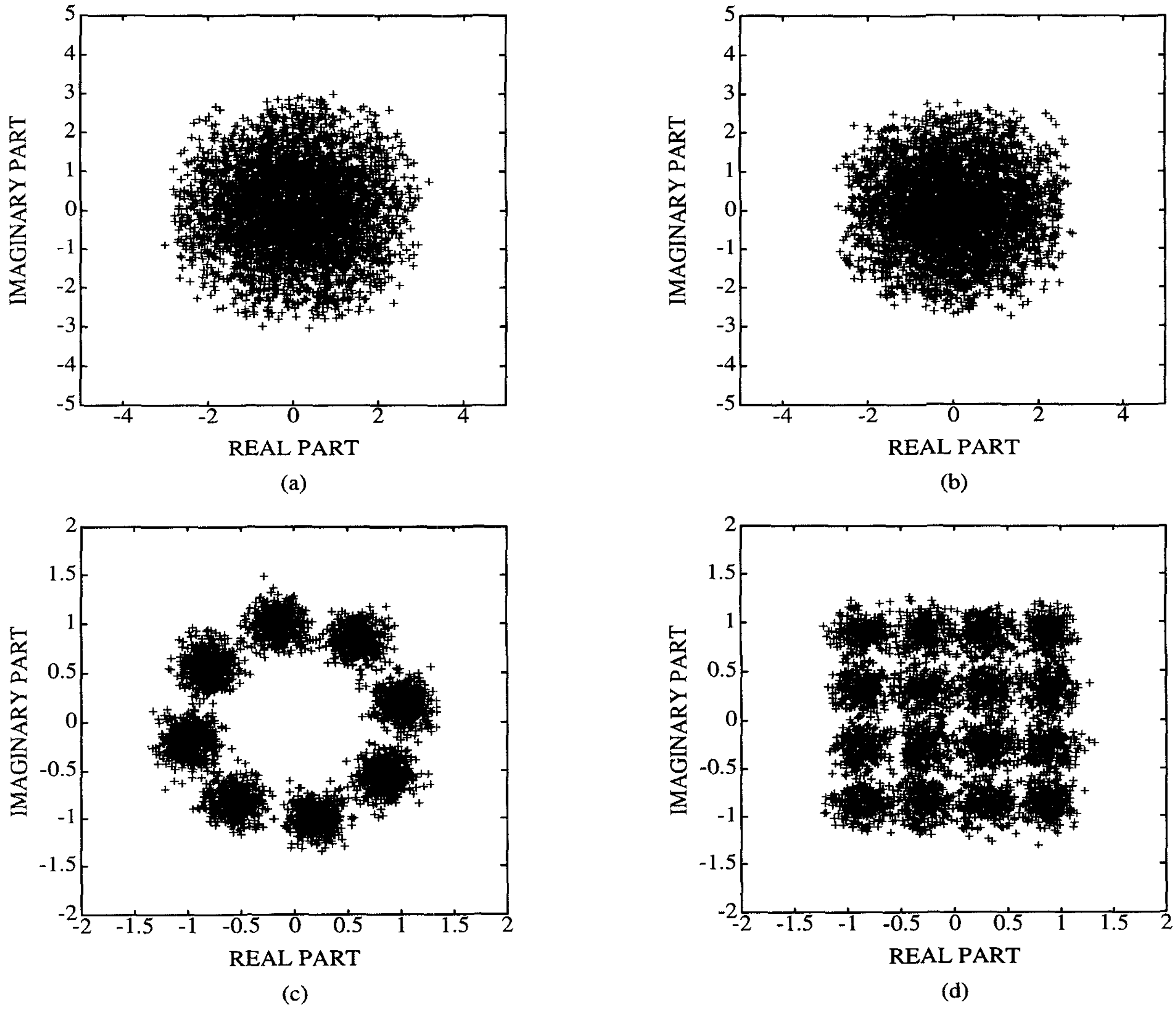

Figure 1. (a) and (b) Unequalized signal constellations associated with $x_1(n)$ and $x_2(n)$, respectively, for $n = 0 \sim 4095$ and SNR = 20 dB; (c) and (d) equalized signal constellations associated with $e_1(n)$ and $e_2(n)$, respectively, for $n = 0 \sim 4095$ using the MS criteria with $(s, l_1, l_2) = (1, 2, 2)$.